Influence of the pair coherence on the charge tunneling through a quantum dot connected to a superconducting lead

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We analyze the charge transport through a single level quantum dot coupled to a normal (N) and superconducting (S) leads where the electron pairs exist either as the coherent (for temperatures below T_c) or incoherent objects (in a region $T_c < T < T^*$). This situation can be achieved in practice if one uses the high T_c superconducting material where various precursor effects have been observed upon approaching T_c from above. Without restricting to any particular microscopic mechanism we investigate some qualitative changes of the nonequilibrium charge current caused by the electron pair coherence.

I. INTRODUCTION

It is a firm experimental fact that the phase transition from normal to superconducting states of the underdoped high T_c materials is accompanied by appearance of a pseudogap [1]. While approaching the critical temperature T_c from above the single particle states are gradually depleted in a certain energy region $|\omega| < \Delta_{pg}$ around the Fermi level. This phenomenon is often interpreted theoretically as a precursor of the true superconducting gap which is usually predicted by the BCS-like treatments for temperatures $T < T_c$. On a microscopic basis the pseudogap can be assigned to presence of the electron pairs. Above T_c their long-range coherence is destroyed by the strong quantum fluctuations arising either from the reduced dimensionality, or due to close neighborhood of the Mott insulating state or because of a competition with various kinds of ordering in the system. The incoherent electron pairs have been unambiguously detected experimentally above T_c in measurements of the large Nernst coefficient [2] and in observation of the Berezinski-Kosterlitz-Thouless phase fluctuations [3]. There is however a great amount of controversy regarding the temperature extent where the incoherent pairs eventually exist. According to available experimental data their existence is established for at least a dozen Kelvin above T_c but such region can perhaps spread over a much wider regime up to T^* at which the pseudogap finally closes.

Various tunneling techniques have been used for a long time as a useful tool for probing the single particle spectra of the correlated systems. Recent technological progress of the spectroscopic methods such as the STM [4], the **k**-resolved photoemission (ARPES) [5], the Andreev-type techniques [6] and the Fourier transformed scanning tunneling spectroscopy [7] allow for a precise measurements of the energy, momentum and space dependent density of states. In the present work we propose to consider the N-S junction with the quantum dot located in between. In this setup one would be able not only to detect the (pseudo)gap in the single particle excitation spectrum but, due to activation of additional transport channels (mainly the Andreev reflections), there would be a possibility to analyze the electron pair coherence.

Our discussion here is not limited to any particular microscopic model describing the electron pair formation and their eventual coherence. On rather general grounds we investigate the proximity effect which leads to a particle-hole mixing at small energies $|\omega| \leq \Delta$ and furthermore we analyze its influence on the effective charge transport through the quantum dot. In section II we briefly introduce the problem, in particular explaining how we treat the coherent and incoherent electron pairs. Next, we show the QD spectrum without any correlations (section III) and for the limit of very strong correlations (section IV). The main part of our study is in the section V, where we discuss the differential conductance as a function of the applied bias V and temperature T, both below and above T_c.

II. FORMULATION OF THE PROBLEM

For a description of the quantum dot (QD) coupled to a normal (N) and superconducting (S) leads we consider the single impurity Anderson model

$$\begin{aligned} \hat{H} &= \hat{H}_N + \hat{H}_S + \sum_{\sigma} \epsilon_d \hat{d}^{\dagger}_{\sigma} \hat{d}_{\sigma} + U \, \hat{n}_{d\uparrow} \hat{n}_{d\uparrow} \\ &+ \sum_{\mathbf{k},\sigma} \sum_{\beta=\{N,S\}} \left[V_{\mathbf{k}\beta} \, \hat{d}^{\dagger}_{\sigma} \hat{c}_{\mathbf{k}\beta\sigma} + \text{h.c.} \right]. \end{aligned} \tag{1}$$

Operators d_σ ($d^\dagger_\sigma$) annihilate (create) the QD electrons whose energy is ε_d. The Coulomb potential $U > 0$ describes a repulsion between electrons of opposite spin σ and the hybridization $V^*_{\mathbf{k}\beta}$ is responsible for transferring electrons from the QD the normal ($\beta = N$) or superconducting (S) leads.

We assume that the normal electrode is described by the Hamiltonian of noninteracting fermions $\hat{H}_N = \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}N} - \mu_N)\, \hat{c}^{\dagger}_{\mathbf{k}\sigma N} \hat{c}_{\mathbf{k}\sigma N}$. For description of the superconducting lead we use a general expression

$$\hat{H}_S = \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}S} - \mu_S)\, \hat{c}^{\dagger}_{\mathbf{k}\sigma S} \hat{c}_{\mathbf{k}\sigma S} + \hat{V}_{pairing}, \tag{2}$$

where the two-body term $\hat{V}_{pairing}$ induces either the coherent (for $T < T_c$) or incoherent electron pairs (above

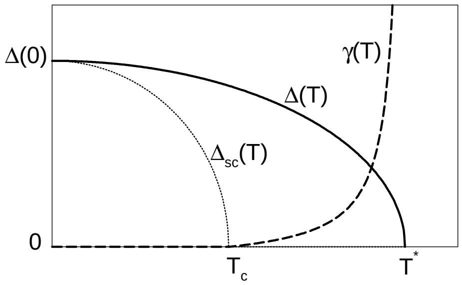

FIG. 1: Temperature dependence of the single particle gap $\Delta(T)$ (solid line), the damping rate $\gamma(T)$ (dashed line) and the superconducting order parameter $\Delta_{sc}(T)$ (dotted line).

T_c).

Without specifying $\hat{V}_{pairing}$ nor restricting to any particular microscopic mechanism of superconductivity we use the basic BCS-type results. For the superconducting state (below the critical temperature T_c) we introduce, in a standard way, the following retarded Green's function expressed in the Nambu representation

$$G^r_S(\mathbf{k},\omega)^{-1} = \begin{pmatrix} \omega - \xi_{\mathbf{k}S} & \Delta_{\mathbf{k}} \\ \Delta_{\mathbf{k}} & \omega + \xi_{\mathbf{k}S} \end{pmatrix} \tag{3}$$

where $\xi_{\mathbf{k}\beta} = \varepsilon_{\mathbf{k}\beta} - \mu_\beta$ measures the energies from the chemical potential. The off-diagonal terms $\Delta_{\mathbf{k}}$ have as usually a meaning of the gap in the single particle excitation spectrum of S electrons.

Since the excitation gap is known to develop well above the transition temperature we follow the arguments of Levin et al [8] and impose the following phenomenological Ansatz

$$\Delta^2_{\mathbf{k}} = \Delta^2_{\mathbf{k},sc} + \Delta^2_{\mathbf{k},pg}. \tag{4}$$

The first part in (4) is related to the superconducting order parameter $\langle \hat{c}_{-\mathbf{k}\downarrow S} \hat{c}_{\mathbf{k}\uparrow S} \rangle$ while the latter one comes from the pseudogap. For a quantitative discussion we propose the temperature dependence given by

$$\Delta_{\mathbf{k},sc}(T) = \begin{cases} \Delta_{\mathbf{k}}(0)\sqrt{1 - \left(\frac{T}{T_c}\right)^2} & \text{for } T \le T_c, \\ 0 & \text{for } T > T_c. \end{cases} \tag{5}$$

The pseudogap contribution $\Delta_{\mathbf{k},pq}$ to the effective gap (4) origins from the preformed electron pairs. Above T_c their long range coherence is missing (hence a name of the incoherent pairs) and such pairs ultimately dissociate at a certain temperature T^*. In analogy to (5) we propose to consider the following temperature dependence

$$\Delta_{\mathbf{k}}(T) = \Delta_{\mathbf{k}}(0) \sqrt{1 - \left(\frac{T}{T^*}\right)^2}. \tag{6}$$

Absence of the off-diagonal long range order above T_c implies that the retarded Green's function (3) must reduce to a diagonal structure for all temperatures $T_c < T < T^*$. In such a case the pseudogap affects the single particle spectrum only through the BCS-like shape of the diagonal part [8]

$$G^r_S(\mathbf{k},\omega) = \begin{pmatrix} \frac{u^2_{\mathbf{k}}}{\omega - E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} + \frac{v^2_{\mathbf{k}}}{\omega + E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} & 0 \\ 0 & \frac{v^2_{\mathbf{k}}}{\omega - E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} + \frac{u^2_{\mathbf{k}}}{\omega + E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} \end{pmatrix} \tag{7}$$

with the quasiparticle dispersion $E_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}S} + \Delta^2_{\mathbf{k}}}$ and the usual coherence factors $u^2_{\mathbf{k}} = \frac{1}{2}\left(1 + \frac{\xi_{\mathbf{k}S}}{E_{\mathbf{k}}}\right) = 1 - v^2_{\mathbf{k}}$. This sort of behavior (7) can be derived on a microscopic level investigating the pairing interactions beyond the mean-field BCS framework [9].

In addition to (5) and (6) we have chosen for computational purposes some phenomenological damping rate

$$\gamma_{\mathbf{k}} = \begin{cases} 0^+ & \text{for } T \le T_c, \\ \gamma_{\mathbf{k}}(0)\ \frac{T - T_c}{T^* - T} & \text{for } T_c < T \le T^* \end{cases} \tag{8}$$

and assumed its momentum variation in the following way $\gamma_{\mathbf{k}} = \gamma^2/(\gamma + \frac{|\xi_{\mathbf{k}S}|}{1000})$ where parameter $\gamma \equiv \gamma_{\mathbf{k}_F}$. Figure (1) illustrates the temperature dependences described in this section.

III. THE UNCORRELATED QD

To introduce the formalism of our calculations we first start analyzing the equilibrium case $\mu_L = \mu_R$. In the standard Nambu notation we express the retarded Green's function of the QD through the Dyson equation

$$G^r_d(\omega)^{-1} = g^r_d(\omega)^{-1} - \Sigma^r_d(\omega) \tag{9}$$

with two contributions to the matrix selfenergy $\Sigma^r_d(\omega) = \Sigma^r_N(\omega) + \Sigma^r_S(\omega)$. This problem can be solved exactly in a case of the noninteracting QD ($U = 0$) when

$$g^{0r}_d(\omega)^{-1} = \begin{pmatrix} \omega - \varepsilon_d + i0^+ & 0 \\ 0 & \omega + \varepsilon_d + i0^+ \end{pmatrix} \tag{10}$$

and the selfenergies simplify to [10]

$$\Sigma^{0r}_N(\omega) = \begin{pmatrix} \sum_{\mathbf{k}} \frac{|V_{\mathbf{k}N}|^2}{\omega - \xi_{\mathbf{k}N} + i0^+} & 0 \\ 0 & \sum_{\mathbf{k}} \frac{|V_{\mathbf{k}N}|^2}{\omega + \xi_{\mathbf{k}N} + i0^+} \end{pmatrix} \tag{11}$$

$$\Sigma^{0r}_S(\omega) = \sum_{\mathbf{k}} |V_{\mathbf{k}S}|^2 \times \tag{12}$$
$$\begin{pmatrix} \frac{u^2_{\mathbf{k}}}{\omega - E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} + \frac{v^2_{\mathbf{k}}}{\omega + E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} & \frac{u_{\mathbf{k}} v_{\mathbf{k}}}{\omega + E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} - \frac{u_{\mathbf{k}} v_{\mathbf{k}}}{\omega - E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} \\ \frac{u_{\mathbf{k}} v_{\mathbf{k}}}{\omega + E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} - \frac{u_{\mathbf{k}} v_{\mathbf{k}}}{\omega - E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} & \frac{v^2_{\mathbf{k}}}{\omega - E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} + \frac{u^2_{\mathbf{k}}}{\omega + E_{\mathbf{k}} + i\gamma_{\mathbf{k}}} \end{pmatrix}.$$

The BCS coefficients $u_{\mathbf{k}} v_{\mathbf{k}} = \frac{\Delta_{\mathbf{k},sc}}{2E_{\mathbf{k}}}$ assure that a proximity effect in the QD occurs only for temperatures $T < T_c$. Otherwise, the pseudogap appearing in the spectrum of S electrons affects the Green's function (7) via the diagonal parts of (12).

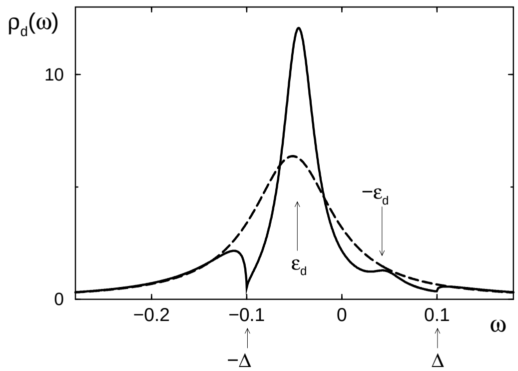

FIG. 2: Spectral function $\rho_d(\omega)$ of the QD for $U=0$ with the right h.s. electrode being in a superconducting (solid line) and in a normal state (dashed line). We used the isotropic energy gap $\Delta_{\mathbf{k}} = 0.1D$ and $\varepsilon_d = -0.05D$, $\Gamma_\beta = 0.05D$, $\gamma = 0.01D$ and set the half-bandwidth D as a unit for energies.

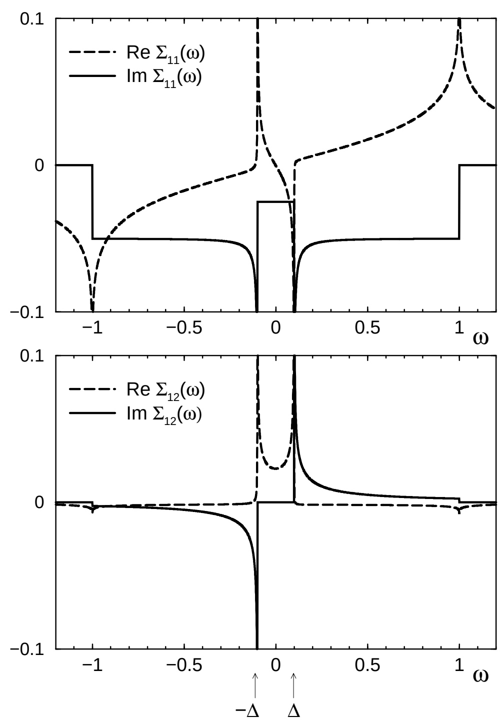

FIG. 3: The real and imaginary parts of the matrix selfenergy for the diagonal $\Sigma^r_{11}(\omega)$ (top panel) and off-diagonal terms $\Sigma^r_{12}(\omega)$ (bottom panel). We used the same set of parameters as in figure 1.

The hybridization coupling $V_{\mathbf{k}\beta}$ can be conveniently replaced by the weighted density function $\Gamma_\beta(\omega) = 2\pi \sum_{\mathbf{k}} |V_{\mathbf{k}\beta}|^2 \delta(\omega - \varepsilon_{\mathbf{k}\beta})$. For both electrodes being normal the QD spectral function $\rho_d(\omega) = -\frac{1}{\pi} G^r_{d\,11}(\omega)$ acquires a Lorentzian shape centered around the single particle level ε_d (see the dashed line in figure 2) with the effective broadening $\Gamma = \Gamma_N + \Gamma_S = 2\Gamma_N$.

If one of the electrodes is a superconductor with an isotropic ($\mathbf{k}$ independent) energy gap we can notice some qualitative (and quantitative) differences in the QD spectrum.

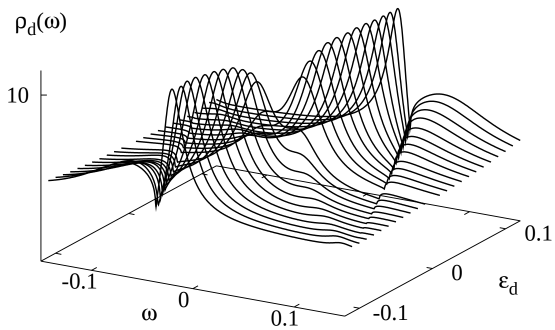

FIG. 4: The ground state spectral function $\rho_d(\omega)$ of the QD for $U=0$ versus a varying position of the energy level ε_d. One can note a clear particle-hole mixing (two Lorentians built around $\pm\varepsilon_d$).

(i) Since S electrons can occupy no states in the energy gap $|\omega| < \Delta$ thereby the line broadening gets reduced by 50% and in consequence the QD peak around ε_d becomes more pronounced.

(ii) A large amount of S electron states is cumulated near $\omega = \pm\Delta$ (i.e. at the square root divergences in the density of the S lead). Efficiency of the hybridization $V_{\mathbf{k}S}$ is there considerably enhanced leading to a depletion of the QD states at $\omega = \pm\Delta$.

(iii) A role of well defined quasiparticles in the superconducting state is played by the electron pairs. Due to the hybridization $V_{\mathbf{k}S}$ such a particle hole mixing is also transfered onto the QD spectrum (the proximity effect). Indeed, in figures 2 and 4 we notice that besides the Lorentzian peak centered around ε_d there also appears its tiny mirror reflection at $-\varepsilon_d$.

To provide the arguments for the above mentioned effects we plot in figure 3 the diagonal and off-diagonal parts of the matrix selfenergy $\Sigma_d(\omega)$. An odd symmetry of $\mathrm{Im}\,[\Sigma_{12}(\omega)]$ gives a non-vanishing $\mathrm{Re}\,[\Sigma_{12}(\omega)]$ for all energies located inside the energy gap $|\omega| < \Delta$. In diagonal term $\Sigma_{11}(\omega)$ the imaginary part is even (and negative) while the real part is odd (therefore vanishing at $\omega = 0$). Similar quantitative behavior of the matrix selfenergy $\Sigma_d(\omega)$ has been previously reported by several authors [12, 13, 14, 15], however, no clear evidence of the particle-hole mixing has been emphasized so far.

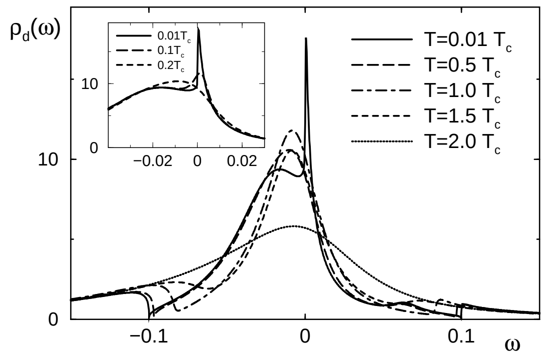

FIG. 5: Spectral function $\rho_d(\omega)$ of the QD in the limit $U=\infty$ obtained for $\varepsilon_d = -0.05D$, $\Gamma_\beta = 0.01D$ assuming the isotropic energy gap $\Delta_{\mathbf{k}}(T=0) = 0.1D$ and $T^* = 2T_c$.

IV. THE STRONG CORRELATION LIMIT

It is well known both from the theoretical [11] and experimental studies [16, 17] that the Coulomb interactions have a remarkable influence on transport properties through the QD. In particular, such correlations are responsible for the Coulomb blockade (observed by oscillations of the differential conductance) and, at sufficiently low temperatures, produce the Kondo resonance leading to enhancement of the conductance to the unitary limit value $2e^2/h$.

In this section we consider the correlations focusing on the extreme limit of $U = \infty$. Under such condition no double occupancy of the QD is allowed and one expects it to have a tremendous effect on the charge tunneling especially in the anomalous channels involving the electron pairs.

Excluding the doubly occupied states from the Hilbert space can be formally achieved using the auxiliary fields

$$\hat{d}_\sigma^\dagger = \hat{f}_\sigma^\dagger \hat{b} \qquad \hat{d}_\sigma = \hat{b}^\dagger \hat{f}_\sigma \tag{13}$$

where the boson $\hat{b}^{(\dagger)}$ and fermion operators $\hat{f}_\sigma^{(\dagger)}$ correspond to an annihilation (creation) of the empty and singly occupied states on the QD. These new fields must obey the local constraint $\hat{b}^\dagger \hat{b} + \sum_\sigma \hat{f}_\sigma^\dagger \hat{f}_\sigma = 1$.

There are various methods to deal with the local constraint. For simplicity, we apply here the technique proposed by Le Guillou and Ragoucy [18] where projecting out of the doubly occupied states is achieved by appropriate commutation relations between the operators of auxiliary fields. For the present context (1) some necessary technical details have been previously discussed in Ref. [10].

In the limit $U = \infty$ the Dyson equation (9) can be solved using the renormalized propagator $g_d^r(\omega) = (1 - n_{-\sigma})\, g_d^{r0}(\omega)$ and the matrix selfenergy

$$\Sigma_\beta^r(\omega) = \left[\Sigma_\beta^{0r}(\omega) + \Sigma_\beta^{Ir}(\omega)\right] / \left(1 - n_{-\sigma}\right) \tag{14}$$

where the contribution Σ_β^{0r} of noninteracting electrons is given in (11, 12). The other contribution Σ_β^{Ir} originates from the correlations and under appropriate conditions leads to the Kondo effect [11]. One finds [10]

$$\Sigma_\beta^{Ir}(\omega) = n_{\mathbf{k}\beta}\; \tau_3\; \Sigma_\beta^{0r}(\omega)\; \tau_3 \tag{15}$$

where τ_3 is the Pauli matrix and $n_{\mathbf{k}\beta}$ denotes an average occupancy of the $\mathbf{k}$-momentum in the β-th lead given by

$$n_{\mathbf{k}\beta} = \begin{cases} \left[1 + \exp(\frac{\xi_{\mathbf{k}N}}{k_B T})\right]^{-1} & \text{for } \beta = N, \\ \frac{1}{2}\left[1 - \frac{\xi_{\mathbf{k}S}}{E_{\mathbf{k}}} \tanh\left(\frac{E_{\mathbf{k}}}{2k_B T}\right)\right] & \text{for } \beta = S. \end{cases} \tag{16}$$

In figure 5 we show the spectral function $\rho_d(\omega)$ calculated for several temperatures in both, the superconducting and pseudogap states. In a comparison to the previous situation $U=0$ we can notice that:

(i) for temperatures $T < T_c$ there are visible two Lorentzian peaks, however their positions are more shifted from $\pm\varepsilon_d$ because of a finite real part of the matrix selfenergy (14),

(ii) for very low temperatures ($T < T_K$) there appears a narrow Kondo resonance at the Fermi energy associated with the spin singlet made of the QD and itinerant electrons (see the inset in Fig. 5),

(iii) in the pseudogap regime above T_c we no longer observe a tiny Lorentzian at $\omega \simeq -\varepsilon_d$ and simultaneously a dip of the spectral function at $|\omega| = \pm\Delta_{pg}(T)$ gets smeared because of the damping effects.

For the particular set of parameters $\Gamma_\beta = 0.01D$, ε_d used in figure 5 we estimate the Kondo peak disappears for temperatures higher than $T_K \simeq 0.15T_c$.On the other hand, for temperature exceeding T^* the QD spectrum evolves back to its single Lorentzian peak centered around ε_d.

V. TRANSPORT PROPERTIES

In order to study the nonequilibrium physics we use the Keldysh formalism. Applying a bias V leads to imbalance of the chemical potentials $\mu_N - \mu_S = eV$ which induces the charge current $J(V) = -e\frac{d}{dt}\sum_{\mathbf{k},\sigma}\langle c_{\mathbf{k}N\sigma}^\dagger c_{\mathbf{k}N\sigma}\rangle$ through the QD.

Following the procedure described previously [10] we express the charge current $J(V)$ in terms of the following contributions

$$J = J_{11} + J_{12} + J_{22} + J_A. \tag{17}$$

The first three components in equation (17) have the Landauer-type structure

$$J_{ij}(V) = \frac{2e}{h}\int d\omega\; T_{ij}(V)\left[f(\omega - eV) - f(\omega)\right] \tag{18}$$

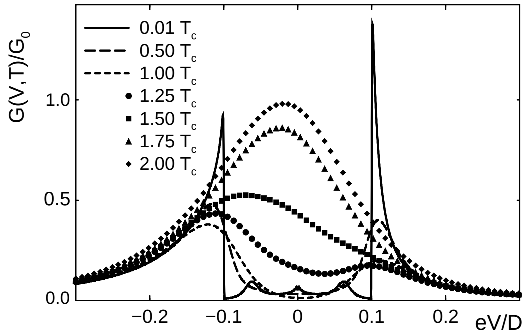

FIG. 6: The differential conductance $G(V,T)$ versus the applied bias V for a representative set of temperatures in the superconducting region (the lines) and for the pseudogap phase (the symbols). We used the same set of parameters as in figure 5 and conductnce is expressed in units of $G_0 \equiv G(0,T^*)$.

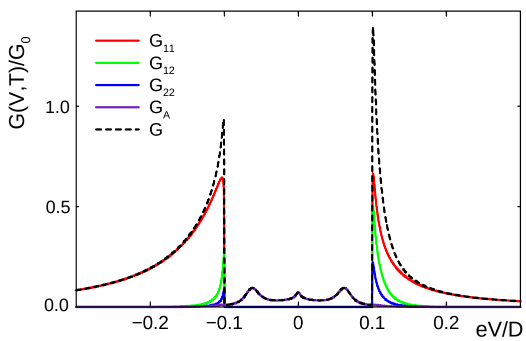

FIG. 7: Contributions of the normal G_{11} (the red line) and the anomalous channels G_{12} (blue), G_{22} (green), G_A (indigo) to the total differential conductance $G(V,T)$. Temperature is $T = 0.01T_c$ (i.e. $T < T_K$) and other parameters have the same values as in figure 5.

where the corresponding transmittance are defined by

$$T_{11}(V) = -\mathrm{Im}\Sigma^r_{11,S}\ |G_{11}|^2\ \Gamma_N(\omega), \quad (19)$$

$$T_{12}(V) = -2\mathrm{Im}\Sigma^r_{12,S}\ \mathrm{Re}\left[G_{11}G^*_{12}\right]\ \Gamma_N(\omega), \quad (20)$$

$$T_{22}(V) = -\mathrm{Im}\Sigma^r_{22,S}\ |G_{12}|^2\,\Gamma_N(\omega). \quad (21)$$

The last contribution describes the Andreev current

$$J_A(V) = \frac{2e}{h}\int d\omega\ T_A(V)\left[f(\omega - eV) - f(\omega + eV)\right] \quad (22)$$

with

$$T_A(V) = -\mathrm{Im}\Sigma^r_{22,N}\ |G_{12}|^2\,\Gamma_N(\omega). \quad (23)$$

This kind of current arises when electron from the N lead is converted into the Copper pair in the S electrode and simultaneously a hole is reflected back to the N lead. For a detailed discussion of such anomalous Andreev current see for instance the recent review article [6]. In a case when the both leads are normal (i.e. $\Delta \to 0$) there survives only $J_{11}(V)$ current and moreover the transmittance (19) simplifies to the usual form $T(\omega) = \rho_d(\omega)\ \frac{\Gamma_N(\omega)\Gamma_s(\omega)}{\Gamma_N(\omega)+\Gamma_S(\omega)}$ [11].

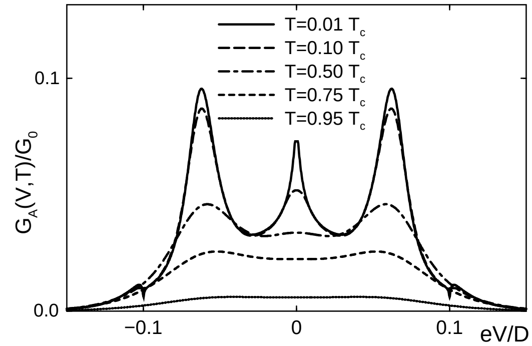

FIG. 8: The differential conductance $G_A(V,T)$ of the Andreev current for a number of temperatures (see the legend). The zero bias enhancement (central peak) is due to the Kondo resonance. Above T_c the Andreev current vanishes [6].

In figure 6 we plot the differential conductance $G(V,T) = dJ(V)/dV$ as a function of the external bias V for several representative temperatures. In the superconducting state we clearly notice a strong suppression of the charge current at small voltages $|eV| \leq \Delta(T)$. Due to energy gap in the spectrum of S electrons the only possible process of a charge tunneling occurs then through the Andreev channel. At temperatures below T_c the non-vanishing conductance $G(V,T) \simeq G_A(V,T)$ for $|eV| \leq \Delta(T)$ is almost an order of magnitude smaller than the normal state conductance $G(V,T^*)$.

For sufficiently low temperatures $T < T_K$, we observe formation of the Kondo peak (see figure 5) which affects the differential conductance at small bias $|V|$. In the present it gives some enhancement of the zero bias conductance (see the inset of figure 9) through the Andreev scattering. This zero bias anomaly is however rather residual as compared to the N-QD-N junctions [11]. A strong suppression of the zero bias anomaly has been previously pointed out by several authors [12]. We checked that the low temperature differential conductance $G(0,T)$ fits very well the universal parabolic variation characteristic for the Kondo regime.

For higher voltages, exceeding $\Delta(T)$, the dominant role to the charge transport comes from the normal current $J_{11}(V)$. In figure 7 we show each of the contributions to the total conductance for a small temperature $T < T_K < T_c$. The anomalous channels $J_{12}(V)$ and $J_{22}(V)$ get activated outside the energy gap. But these contributions, as well as $G_A(V)$, do quickly diminish for an increasing bias $|V|$.

The in-gap conductance arising from the Andreev current is very sensitive to the temperature (see figure 8). Already for temperatures higher then $T \sim T_K$ the Kondo peak is washed out which gives a concomitant disappearance of the zero bias anomaly. Upon further increasing the temperature there occurs a gradual suppression of the Andreev current which completely vanishes at $T \to T_c^-$.

For temperatures above T_c the charge current is transmitted only via the normal $J_{11}(V)$ channel. With an

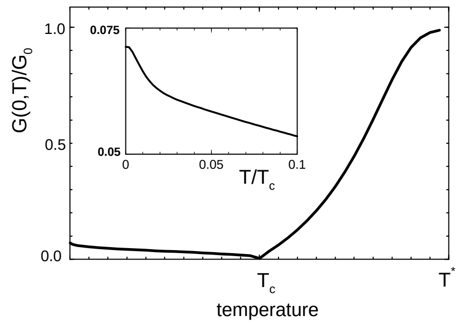

FIG. 9: Temperature dependence of the zero bias differential conductance $G(V=0,T)$ for the set of parameters used in figure 5. Inset shows the enhancement at low temperatures which is due to the Kondo resonance.

increasing temperature the pseudogap becomes gradually filled in (see figure 6) therefore the zero bias conductance smoothly increases, reaching a local maximum at T^*. From basic considerations [11] it is well known that in the normal state (for $T > T^*$) the differential conductance exponentially decreases with respect to T. We would like to stress that the non-monotonous temperature variation of the zero bias differential conductance $G(0,T)$ (shown in figure 9) could be a useful method for identifying the temperature region of the incoherent electron pairs.

VI. CONCLUSIONS

We have investigated the charge tunneling from the normal (N) to superconducting (S) leads via the quantum dot (QD). In agreement with the previous studies [12, 13, 14, 15] we find that the long range off-diagonal superconducting order induces the proximity effect in the quantum dot (independently of the on-dot Coulomb interaction U). Besides the main Lorentzian peak near the single particle energy level ε_d there appears another small counter-peak around $-\varepsilon_d$ due to the particle-hole mixing. Such physical situation occurs for all temperatures below T_c. This effect could be practically observed in measurements of the differential conductance upon varying the gate voltage.

In a presence of the strong on-dot correlations U there arise some additional qualitative features. One of them is the Kondo state which occurs at sufficiently low temperatures, usually below 1K [16]. Spectral function of the QD develops then a narrow peak at the Fermi energy. It has a qualitative influence on the low energy transport properties. Since the normal channel $J_{11}(V)$ is forbidden for $|eV| < \Delta$ (because of the energy gap of S electrons) the only possibility for the charge transfer is through the Andreev tunneling. A magnitude of the Andreev current is by far smaller than the normal charge current but nevertheless, in analogy to the N-QD-N Kondo anomaly [11, 16], we find some enhancement of the zero bias conductance for the N-QD-S setup. This anomaly shows up at temperatures $T < T_K$ but the conductance does not reach the unitary limit value $2e^2/h$.

We have also extended our study on the pseudogap state and found that incoherent electron pairs have a remarkable influence on the charge transport. Above T_c the zero bias differential conductance $G(0,T)$ systematically increases upon increasing temperature because the pseudogap is gradually filled in. Finally, at T^*, it reaches a local maximum and for higher temperatures (in the normal state) has a changeover to the exponential decrease versus T. Such nonmonotonus variation of the zero bias conductance (figure 9) can serve as a sensitive method to identify the temperature region $T_c < T < T^*$ of the incoherent electron pairs.

Some other effects which might eventually come into the play. One of them is a strong anisotropy of the gap function $\Delta_{\mathbf{k}}$ well known for all the HTSC cuprates. To some extent this issue has been recently addressed by ETH group [19]. Authors investigated the conductance of charge current flowing between the conducting STM tip and the quasi-2D superconducting CuO_2 planes involving the apical oxygen atoms (to be regarded as the QD). They focused on studying the inelastic scattering caused by the oxygen atoms' vibrations. In the future work we shall discuss the normal and anomalous currents for such anisotropic gap $\Delta_{\mathbf{k}}$ of the d-wave symmetry. As known from the literature the gapless superconductivity does not preclude appearance of the Kondo resonance [20, 21].

VII. ACKNOWLEDGMENTS

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[1] D.N. Basov and T. Timusk, Rev. Mod. Phys. **77**, 721 (2005); T. Timusk and B. Statt, Rep. Prog. Phys. **62**, 61 (1999).
[2] Y. Wang, L. Li, M. Naughton, G. Gu, S. Uchida, and N.P. Ong, Phys. Rev. Lett. **95**, 247002 (2005).
[3] J. Corson, L. Mallozzi, J. Orenstein, J. Eckstein, and I. Bozovic, Nature **398**, 221 (1999).
[4] O. Fisher, M. Kugler, I. Maggio-Aprile, Ch. Berthod and Ch. Renner, cond-mat/0610672.
[5] A. Damascelli, Z. Hussain, and Z.-X. Shen, Rev. Mod. Phys. **75**, 473 (2003).
[6] G. Deutscher, Rev. Mod. Phys. **77**, 109 (2005).
[7] K. McElroy, J. Lee, J.A. Slezak, D.-H. Lee, H. Eisaki, S. Uchida, and J.C. Davis, Science **309**, 1048 (2005).
[8] Q. Chin, J. Stajic, S. Tan, K. Levin, Phys. Rep. **412**, 1 (2005).

[9] J.M. Singer, M.H. Pedersen, T. Schneider, H. Beck and H.-G. Matuttis, Phys. Rev. B **54**, 1286 (1996); Y.M. Vilk and A.-M.S. Tremblay, J. Phys. I France **7**, 1309 (1997); T. Domański and J. Ranninger, Phys. Rev. Lett. **91**, 255301 (2003).

[10] M. Krawiec and K.I. Wysokiński, Supercond. Sci. Technol. **17**, 103 (2004).

[11] Y. Meir and N.S. Wingreen, Phys. Rev. Lett. **67**, 2512 (1992); Y. Meir, N.S. Wingreen, P.A. Lee, Phys. Rev. Lett. **70**, 2601 (1993).

[12] R. Fazio and R. Raimondi, Phys. Rev. Lett. **80**, 2913 (1998); Phys. Rev. Lett. **82**, 4950 (1998); P. Schwab and R. Raimondi, Phys. Rev. B **59**, 1637 (1999).

[13] Q.-F. Sun, J. Wang, and T.-H. Lin, Phys. Rev. B **59**, 3831 (1999).

[14] A.A. Clerk, A. Ambegaokar, and S. Herschfield, Phys. Rev. B **61**, 3555 (2000); A.A. Clerk, A. Ambegaokar, Phys. Rev. B **61**, 9109 (2000).

[15] J.C. Cuevas, A. Levy Yeyati, and A. Martin-Rodero, Phys. Rev. B **63**, 094515 (2001).

[16] D. Goldhaber-Gordon, H. Shtrikman, D. Mahalu, D. Abusch-Magder, U. Mairav, and M.A. Kastner, Nature **391**, 156 (1998).

[17] S. De Franceschi, R. Hanson, W.G. van der Wiel, J.M. Elzerman, J.J. Wijpkema, T. Fujisawa, S. Tarucha, L.P. Kouwenhoven, Phys. Rev. Lett. **89**, 156801 (2002).

[18] J.C. Le Guillou and E. Ragoucy, Phys. Rev. B **52**, 2403 (1995).

[19] S. Pilgram, T.M. Rice, and M. Sigrist, Phys. Rev. Lett. **97**, 117003 (2006).

[20] L.S. Borkowski and P.J. Hirschfeld, Phys. Rev. B **46**, 9274 (1992).

[21] M. Vojta and R. Bulla, Phys. Rev. B **65**, 014511 (2001); M. Vojta, R. Zitzler, R. Bulla, and Th. Pruschke, Phys. Rev. B **66**, 134527 (2002); A. Polkovnikov, S. Sachdev, and M. Vojta, Phys. Rev. Lett. **86**, 296 (2001).